**Lecture 2**

2023/2024

# Microwave Devices and Circuits for Radiocommunications

# 2023/2024

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday 16-18, ~~Online~~, P8
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 20-27.02.2024 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

# 2023/2024

- Laboratory – **associate professor Radu Damian**
  - Tuesday 08-12, II.13 / (08:10)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 20.02.2024)
    - personal homework

# Materials

- http://rf-opto.etti.tuiasi.ro

Laboratorul de Microunde si Opt

Not secure | rf-opto.etti.tuiasi.ro/microwave_cd.php?chg_lang=0

Main **Courses** Master Staff Research Students Admin

**Microwave CD** Optical Communications Optoelectronics Internet Antennas Practica Networks Educational software

## Microwave Devices and Circuits for Radiocommunications (English)

### Course: MDCR (2017-2018)

**Course Coordinator:** Assoc.P. Dr. Radu-Florin Damian
**Code:** EDOS412T
**Discipline Type:** DOS; Alternative, Specialty
**Credits:** 4
**Enrollment Year:** 4, **Sem.** 7

### Activities

**Course:** Instructor: Assoc.P. Dr. Radu-Florin Damian, 2 Hours/Week, Specialization Section, Timetable:
**Laboratory:** Instructor: Assoc.P. Dr. Radu-Florin Damian, 1 Hours/Week, Group, Timetable:

### Evaluation

Type: **Examen**

**A:** 50%, (Test/Colloquium)
**B:** 25%, (Seminary/Laboratory/Project Activity)
**D:** 25%, (Homework/Specialty papers)

### Grades

Aggregate Results

### Attendance

Course
Laboratory

### Lists

Bonus-uri acumulate (final)
Studenti care nu pot intra in examen

### Materials

**Course Slides**

MDCR Lecture 1 (pdf, 5.43 MB, en, )
MDCR Lecture 2 (pdf, 3.67 MB, en, )
MDCR Lecture 3 (pdf, 4.76 MB, en, )
MDCR Lecture 4 (pdf, 5.58 MB, en, )

RF-OPTO

English | Romana |

Main Courses Master Staff Rese

Grades Student List **Exams** Photos

## Online Exams

In order to participate at online exams you must get ready following

# Site

RF-OPTO

English | Romana |

Main Courses Master Staff Research Students

## Microwave and Optoelectronics Laboratory

We are enlisted in the Telecommunications Department of the Electronics, Telecommunication and Information Technology Faculty (**ETTI**) from the "Gh. Asachi" Technical University (**TUIASI**) in Iasi, Romania

We currently cover inside **ETTI** the fields related to:

- Microwave Circuits and Devices
- Optoelectronics
- Information Technology

## Courses

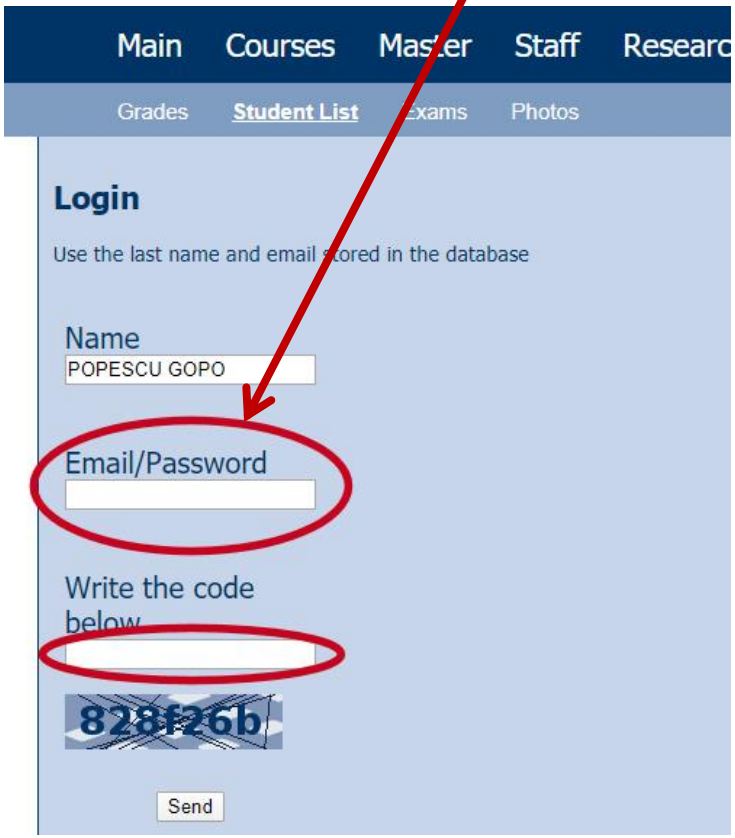
| Nr. | Course | Shortcut | Code | Type | Semester | Credits | Weekly | Examination | Link |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Microwave Devices and Circuits for Radiocommunications | **DCMR** | DOS412T | DOS | 7 | 4 | 0P,1L,0S,2C | Exam | |
| 2 | Monolithic Microwave Integrated Circuits | **CIMM** | RD.IA.207 | DOMS | 11 | 6 | 1.5L,0S,2C,0P | Exam | |
| 3 | Advanced Techniques in the Design of the Radio-communications Systems | **TAPSR** | RD.IA.103 | DIMS | 9 | 6 | 1.5P,0L,0S,2C | Exam | |
| 4 | Optical Communications | **CO** | DOS409T | DOS | 7 | 5 | 0P,1L,0S,3C | Colloquiu | |
| 5 | Optical Communications | **OC** | EDOS409T | DOS | 7 | 5 | 0P,1L,0S,3C | Exam | |
| 6 | Satellite Communications | **CS** | RC.IA.104 | DIMS | 9 | 6 | 0L,0S,2C,1.5P | Exam | |
| 7 | Applied Informatics 1 | **IA1** | DOF135 | DOF | 1 | 4 | 0P,1L,0S,2C | Verificati | |
| 8 | Applied Informatics 1 | **AI1** | EDOF135 | DOF | 1 | 4 | 0P,1L,0S,2C | Verificati | |
| 9 | Databases, Web Programming and Interfacing | **DWPI** | ITT.IA.601 | DIS | 11 | 5 | 1P,1L,0.25S,1C | Verificati | |
| 10 | Web Applications Design | **PAW** | RC.IA.108 | DIMS | 10 | 5 | 1L,0S,1.5C,1P | Exam | |
| 11 | Optoelectronics | **OPTO** | DID405M | DID | 8 | 4 | 0P,1L,0S,2C | Colloquiu | |
| 12 | Microwave Devices and Circuits for Radiocommunications (English) | **MDCR** | EDOS412T | DOS | 8 | 4 | 0P,1L,0S,2C | Exam | |


# Materials

- RF-OPTO
  - **http://rf-opto.etti.tuiasi.ro**
- **David Pozar, "Microwave Engineering"**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by ~~email~~/**online exam > Week4-Week6**
  - used at lectures/laboratory

# Online

- access to **online exams** requires the **password** received by email

# Password

- received by email

Important message from RF-OPTO  Inbox ×

**Radu-Florin Damian**
to me, POPESCU

Romanian > English  Translate message

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

**In atentia: POPESCU GOPO ION**

Parola pentru a accesa examenele pe server-ul **rf-opto** este
Parola: [redacted]

Identificati-va pe server, cu parola, cat mai rapid, pentru confirmare.

**Memorati** acest mesaj intr-un loc sigur, pentru utilizare ulterioara

---

**Attention: POPESCU GOPO ION**

The password to access the exams on the **rf-opto** server is
Password: [redacted]

Login to the server, with this password, as soon as possible, for confirmation.

**Save** this message in a safe place for later use

Reply  Reply all  Forward

| Subject | Correspondents |
|---|---|
| Important message from RF-OPTO | POPESCU GOPO ION |
| Validation of MDCR exam from 02/05/2020 | [redacted] |
| [redacted] | [redacted] |

From Me <rdamian@etti.tuiasi.ro>
Subject **Important message from RF-OPTO**
To [redacted]
Cc Me <rdamian@etti.tuiasi.ro>

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

**In atentia: POPESCU GOPO ION**

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**Attention: POPESCU GOPO ION**

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Password: [redacted]

Login to the server, with this password, as soon as possible, for confirmation.

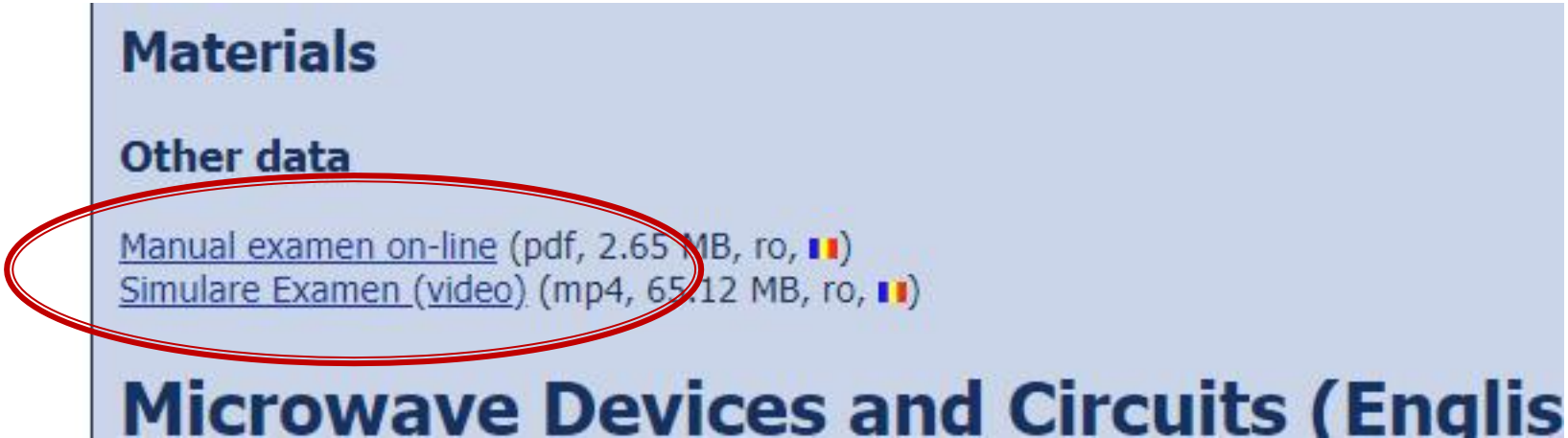
**Save** this message in a safe place for later use

# Adrese email

- Sefii de grupa
  - lista cu adrese de email **utilizate** de toti studentii
    - poate fi @student.etti.tuiasi.ro (@gmail @yahoo etc.)
  - **rdamian@etti.tuiasi.ro**

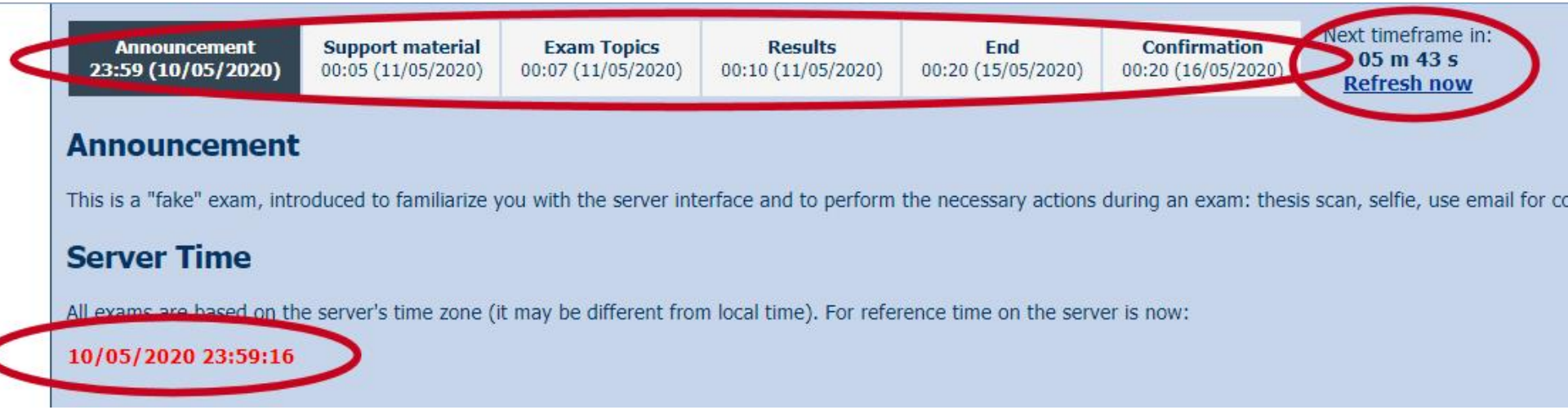
# Online exam manual

- The online exam app used for:
  - ~~lectures (attendance)~~
  - laboratory
  - project
  - ~~examinations~~



# Examen online

- always against a **timetable**
  - long period (lecture attendance/laboratory results)
  - ~~short period (tests: 15min, exam: 2h)~~



# Online results submission

- many numerical values/files

| Schema finala | Rezultate - castig | Rezultate - zgomot | Fisier justificare calcul (factor andrei) | Fisier zap (optional) | T1, fisier parmetri S | T2, fisier parmetri S | Z1 | Z2 | Z3 | Z4 | Z5 | Z6 | Z7 | Ze1 | Zo1 | Ze2 | Zo2 | Ze3 | Zo3 | Ze4 | Zo4 | Ze5 | Zo5 | Ze6 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 86 - 5428 - 259 ... | 86 - 5428 - 260 ... | 86 - 5428 - 261 ... | 86 - 5428 - 316 ... | - | 86 - 5428 - 314 ... | 86 - 5428 - 315 ... | 148.33 | 155.88 | 202.12 | 164.35 | 180.91 | 30.29 | 185.19 | 79.9 | 37 | 68.89 | 45.14 | 61.83 | 45.05 | 57.97 | 46.02 | 61.85 | 45.05 | 68.8 |
| 86 - 5622 - 259 ... | 86 - 5622 - 260 ... | 86 - 5622 - 261 ... | 86 - 5622 - 316 ... | 86 - 5622 - 262 ... | 86 - 5622 - 314 ... | 86 - 5622 - 315 ... | 26.97 | 153.5 | 34.64 | 35.79 | 55.56 | 26.212 | 10.693 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 86 - 5488 - 259 ... | 86 - 5488 - 260 ... | 86 - 5488 - 261 ... | 86 - 5488 - 316 ... | 86 - 5488 - 262 ... | 86 - 5488 - 314 ... | 86 - 5488 - 315 ... | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 86 - 5391 - 259 ... | 86 - 5391 - 260 ... | 86 - 5391 - 261 ... | 86 - 5391 - 316 ... | - | - | - | 50 | 50 | 50 | 50 | 50 | 50 | 50 | 70.14 | 40.39 | 61.85 | 44.59 | 55.7 | 45.2 | 54.89 | 45.38 | 58.65 | 45.8 | 70.0 |
| 86 - 5664 - 259 ... | 86 - 5664 - 260 ... | 86 - 5664 - 261 ... | 86 - 5664 - 316 ... | - | 86 - 5664 - 314 ... | 86 - 5664 - 315 ... | 168.02 | 150.5 | 178.28 | 133.75 | 92.12 | 121.67 | 144.48 | 94.36 | 36.19 | 70.77 | 42.56 | 65.69 | 42.05 | 55.17 | 42.29 | 65.59 | 42.05 | 70.7 |
| 86 - 5665 - 259 ... | 86 - 5665 - 260 ... | 86 - 5665 - 261 ... | 86 - 5665 - 316 ... | - | 86 - 5665 - 314 ... | 86 - 5665 - 315 ... | 162.2 | 80.8 | 209.2 | 140.85 | 135.1 | 183.7 | 167.6 | 94.58 | 36.15 | 78.16 | 39.77 | 65.57 | 45.05 | 65.57 | 45.05 | 78.16 | 39.77 | 94.5 |
| 86 - 5433 - 259 ... | 86 - 5433 - 260 ... | 86 - 5433 - 261 ... | 86 - 5433 - 316 ... | - | 86 - 5433 - 314 ... | 86 - 5433 - 315 ... | 165.138 | 106.228 | 226.157 | 130.134 | 72.71 | 180.177 | 164.616 | 101.36 | 36.11 | 77.22 | 42.49 | 68.02 | 45.62 | 60 | 45.42 | 68.02 | 45.62 | 77.2 |
| 86 - 5608 - 259 ... | 86 - 5608 - 260 ... | 86 - 5608 - 261 ... | 86 - 5608 - 316 ... | - | 86 - 5608 - 314 ... | 86 - 5608 - 315 ... | 150.84 | 152.5 | 30.94 | 32.37 | 54.36 | 19.837 | 29.85 | 64.14 | 40.145 | 54.32 | 46.32 | 53.8 | 46.7 | 53.8 | 46.7 | 54.32 | 46.32 | 54.9 |
| 86 - 5555 - 259 ... | 86 - 5555 - 260 ... | 86 - 5555 - 261 ... | 86 - 5555 - 316 ... | - | 86 - 5555 - 314 ... | 86 - 5555 - 315 ... | 168.001 | 150.288 | 178.399 | 133.115 | 92.491 | 121.257 | 144.126 | 97.05 | 36.16 | 71.13 | 43.09 | 65.45 | 42.12 | 55.66 | 42.18 | 65.45 | 42.12 | 71.1 |

# Online results submission

- many numerical values

| | Z1 | Z2 | Z3 | Z4 | Z5 | Z6 | Z7 |
|---|---|---|---|---|---|---|---|
| | 148.33 | 155.88 | 202.12 | 164.35 | 180.91 | 30.29 | 185.19 |
| | 26.97 | 153.5 | 34.64 | 35.79 | 55.56 | 26.212 | 10.693 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 50 | 50 | 50 | 50 | 50 | 50 | 50 |

√

✗

# Online results submission

Grade = Quality of the work +

+ Quality of the submission

# Exam: Logarithmic scales

$dB = 10 \cdot \log_{10} (P_2 / P_1)$

| | |
|---|---|
| 0 dB | = 1 |
| + 0.1 dB | = 1.023 (+2.3%) |
| + 3 dB | = 2 |
| + 5 dB | = 3 |
| + 10 dB | = 10 |
| -3 dB | = 0.5 |
| -10 dB | = 0.1 |
| -20 dB | = 0.01 |
| -30 dB | = 0.001 |

$dBm = 10 \cdot \log_{10} (P / 1\ mW)$

| | |
|---|---|
| 0 dBm | = 1 mW |
| 3 dBm | = 2 mW |
| 5 dBm | = 3 mW |
| 10 dBm | = 10 mW |
| 20 dBm | = 100 mW |
| -3 dBm | = 0.5 mW |
| -10 dBm | = 100 μW |
| -30 dBm | = 1 μW |
| -60 dBm | = 1 nW |

[dBm] + [dB] = [dBm]

[dBm/Hz] + [dB] = [dBm/Hz]

[x] + [dB] = [x]

# Computing Loss/Gain in circuits

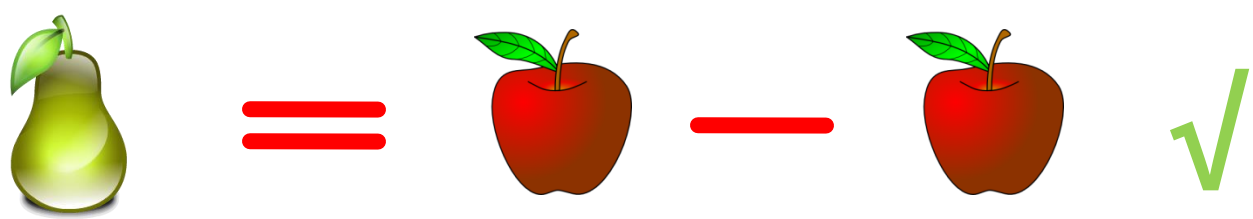
$$\text{Gain/Loss} = \frac{P_{out}}{P_{in}}$$

$$\text{Loss[dB]} = [-]\ 10 \cdot \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

$$\text{Loss[dB]} = [-]\ 10 \cdot \log_{10}\left(\frac{P_{out}}{P_0} \cdot \frac{P_0}{P_{in}}\right)$$

$$\text{Loss[dB]} = [-]\ 10 \cdot \left[\log_{10}\left(\frac{P_{out}}{P_0}\right) - \log_{10}\left(\frac{P_{in}}{P_0}\right)\right]$$

$$\text{Loss[dB]} = [-]\ (P_{out}[\text{dBm}] - P_{in}[\text{dBm}])$$



# Computing Loss/Gain in circuits

$$\text{Loss} = \frac{P_{out}}{P_{in}} < 1$$

$$\text{Loss[dB]} = 10 \cdot \log_{10}\left(\frac{P_{out}}{P_{in}}\right) < 0$$

$$\text{Loss/Attenuation[dB]} = [-]\ 10 \cdot \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

$$\text{Gain} = \frac{P_{out}}{P_{in}} > 1$$

$$\text{Gain[dB]} = 10 \cdot \log_{10}\left(\frac{P_{out}}{P_{in}}\right) > 0$$

$$\text{Attenuation[dB/km]} = \frac{\text{Loss[dB]}}{\text{Length[km]}}$$

# Computing Loss/Gain in circuits

$$\text{Loss/Attenuation} \rightarrow P_{out} < P_{in} \rightarrow P_{out}[\text{dBm}] < P_{in}[\text{dBm}]$$

$$P_{out}[\text{dBm}] = P_{in}[\text{dBm}] - \text{Loss/Attenuation}[\text{dB}]$$

$$\text{Gain/Amplification} \rightarrow P_{out} > P_{in} \rightarrow P_{out}[\text{dBm}] > P_{in}[\text{dBm}]$$

$$P_{out}[\text{dBm}] = P_{in}[\text{dBm}] + \text{Gain/Amplification}[\text{dB}]$$

# Exam: Logarithmic scales

**dB = 10 • $\log_{10}$ ($P_2$ / $P_1$)**

| | |
|---|---|
| 0 dB | = 1 |
| + 0.1 dB | = 1.023 (+2.3%) |
| + 3 dB | = 2 |
| + 5 dB | = 3 |
| + 10 dB | = 10 |
| -3 dB | = 0.5 |
| -10 dB | = 0.1 |
| -20 dB | = 0.01 |
| -30 dB | = 0.001 |

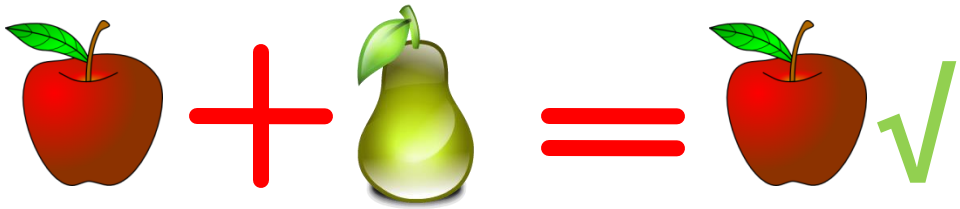
**dBm = 10 • $\log_{10}$ (P / 1 mW)**

| | |
|---|---|
| 0 dBm | = 1 mW |
| 3 dBm | = 2 mW |
| 5 dBm | = 3 mW |
| 10 dBm | = 10 mW |
| 20 dBm | = 100 mW |
| -3 dBm | = 0.5 mW |
| -10 dBm | = 100 µW |
| -30 dBm | = 1 µW |
| -60 dBm | = 1 nW |

**[dBm] + [dB] = [dBm]**

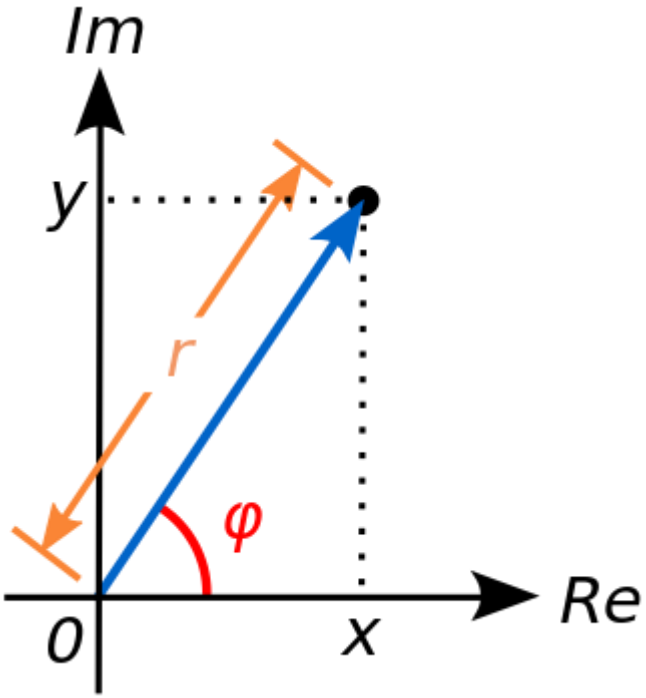
**[x] + [dB] = [x]**

**[dBm/Hz] + [dB] = [dBm/Hz]**



# Examen

- Complex numbers arithmetic!!!!
- $z = a + j \cdot b$ ; $j^2 = -1$



# Polar representation

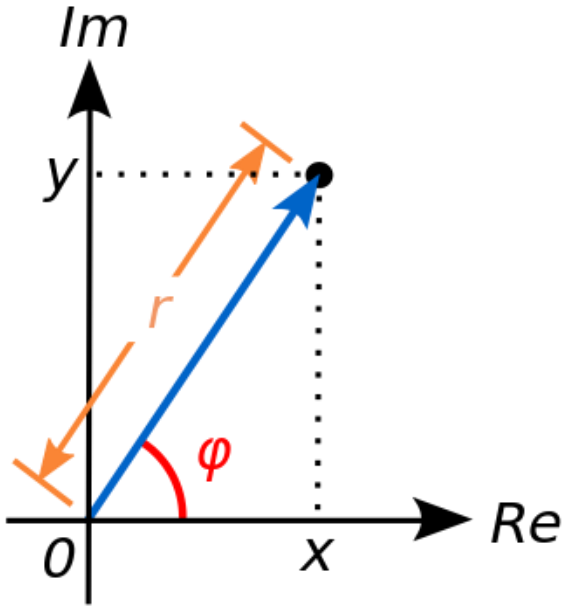
- Euler's formula

$$e^{j\cdot x} = \cos x + j\cdot \sin x; \forall x \in R$$

- Polar representation

$$z = a + j\cdot b = |z|\cdot e^{j\cdot\varphi}$$

$$z = a + j\cdot b = |z|\cdot(\cos\varphi + j\cdot\sin\varphi)$$




$$z^n = \left(|z|\cdot e^{j\cdot\varphi}\right)^n = |z|^n\cdot e^{j\cdot n\cdot\varphi} = |z|^n\cdot\left[\cos(n\cdot\varphi) + j\cdot\sin(n\cdot\varphi)\right]$$

$$\sqrt{z} = \left(|z|\cdot e^{j\cdot\varphi}\right)^{1/2} = \sqrt{|z|}\cdot e^{j\cdot\frac{\varphi}{2}} = \sqrt{|z|}\cdot\left(\cos\frac{\varphi}{2} + j\cdot\sin\frac{\varphi}{2}\right)$$

$$z\cdot w = |z|\cdot e^{j\cdot\varphi}\cdot|w|\cdot e^{j\cdot\theta} = |z|\cdot|w|\cdot e^{j\cdot(\varphi+\theta)} = |z|\cdot|w|\cdot\left[\cos(\varphi+\theta) + j\cdot\sin(\varphi+\theta)\right]$$

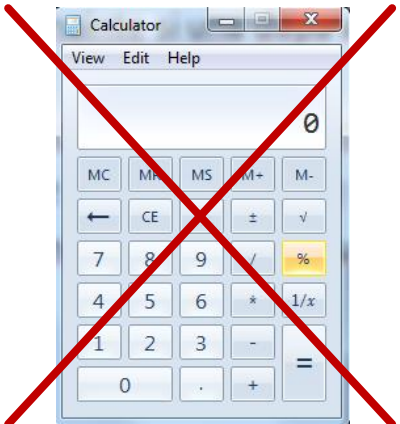
$$z/w = \frac{|z|\cdot e^{j\cdot\varphi}}{|w|\cdot e^{j\cdot\theta}} = \frac{|z|}{|w|}\cdot e^{j\cdot\varphi}\cdot e^{-j\cdot\theta} = \frac{|z|}{|w|}\cdot\left[\cos(\varphi-\theta) + j\cdot\sin(\varphi-\theta)\right]$$

# Polar representation

- **Attention to angle numerical values!!**
  - math software – work in standard unit: radians
    - a **conversion** is necessary before and after using a trigonometric function (sin, cos, tan, atan, tanh)
  - scientific calculators have the built-in option of choosing the angle unit
    - always **double check** current working unit

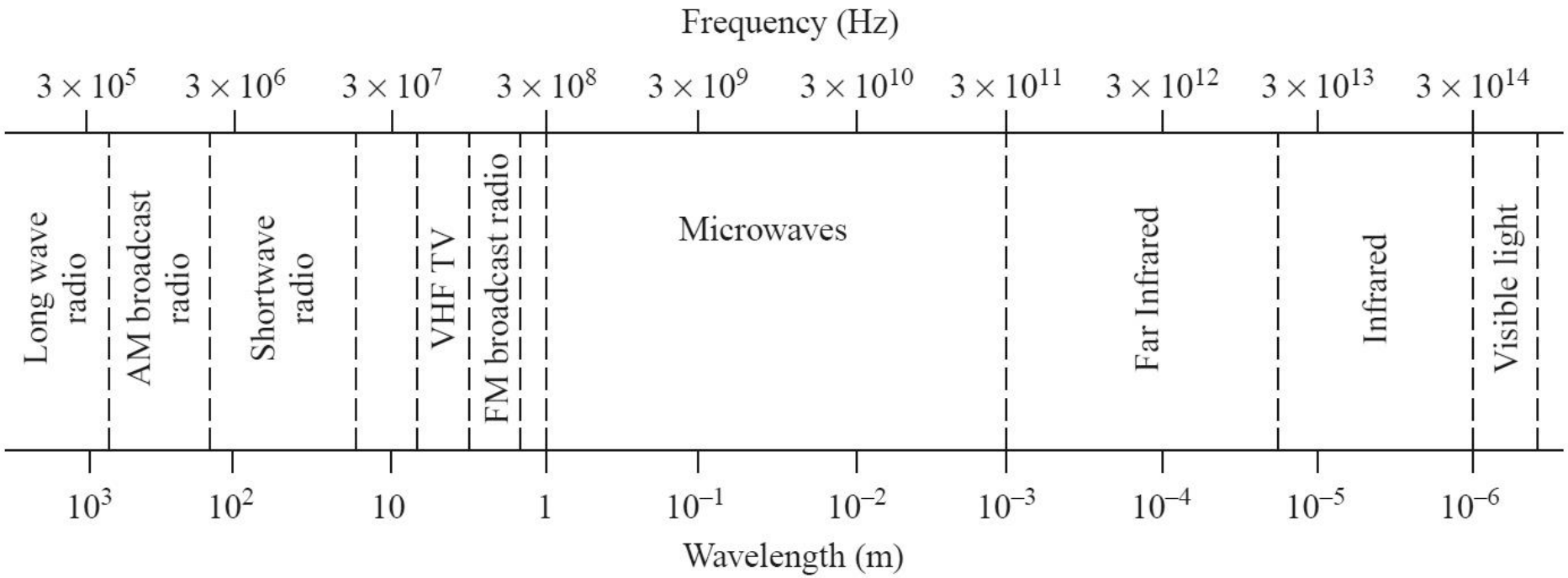
$$\varphi[°] = 180° \cdot \frac{\varphi[rad]}{\pi}$$

$$\varphi[rad] = \pi \cdot \frac{\varphi[°]}{180°}$$




# Introduction

# Microwaves




- typically
  - f ≈ 1÷3GHz – 300GHz
  - λ ≈ 1mm – 10cm

# Microwaves

Typical Frequencies

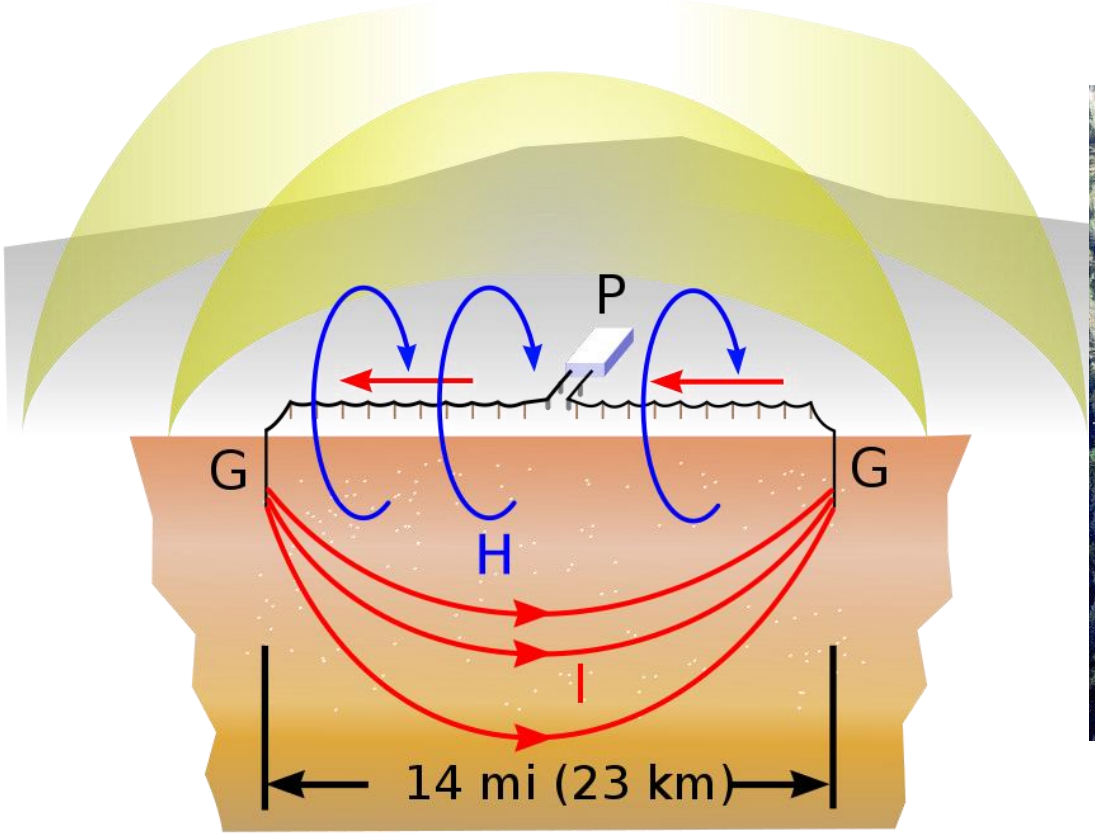
| | |
|---|---|
| AM broadcast band | 535–1605 kHz |
| Short wave radio band | 3–30 MHz |
| FM broadcast band | 88–108 MHz |
| VHF TV (2–4) | 54–72 MHz |
| VHF TV (5–6) | 76–88 MHz |
| UHF TV (7–13) | 174–216 MHz |
| UHF TV (14–83) | 470–890 MHz |
| US cellular telephone | 824–849 MHz |
| | 869–894 MHz |
| European GSM cellular | 880–915 MHz |
| | 925–960 MHz |
| GPS | 1575.42 MHz |
| | 1227.60 MHz |
| Microwave ovens | 2.45 GHz |
| US DBS | 11.7–12.5 GHz |
| US ISM bands | 902–928 MHz |
| | 2.400–2.484 GHz |
| | 5.725–5.850 GHz |
| US UWB radio | 3.1–10.6 GHz |

Approximate Band Designations

| | |
|---|---|
| Medium frequency | 300 kHz–3 MHz |
| High frequency (HF) | 3 MHz–30 MHz |
| Very high frequency (VHF) | 30 MHz–300 MHz |
| Ultra high frequency (UHF) | 300 MHz–3 GHz |
| L band | 1–2 GHz |
| S band | 2–4 GHz |
| C band | 4–8 GHz |
| X band | 8–12 GHz |
| Ku band | 12–18 GHz |
| K band | 18–26 GHz |
| Ka band | 26–40 GHz |
| U band | 40–60 GHz |
| V band | 50–75 GHz |
| E band | 60–90 GHz |
| W band | 75–110 GHz |
| F band | 90–140 GHz |

# ELF, VLF

- Extremely low frequency, 3 - 30 Hz
- Very low frequency, 3 - 30 kHz




# ~ Microwaves

- Electrical Length (Phase Length)
  - l – physical length
  - E = β·l – electrical Length

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left( \frac{l}{\lambda} \right)$$

$$E = \beta \cdot l = \frac{2\pi}{c_0} \cdot \left( l \cdot f \cdot \sqrt{\varepsilon_r} \right)$$

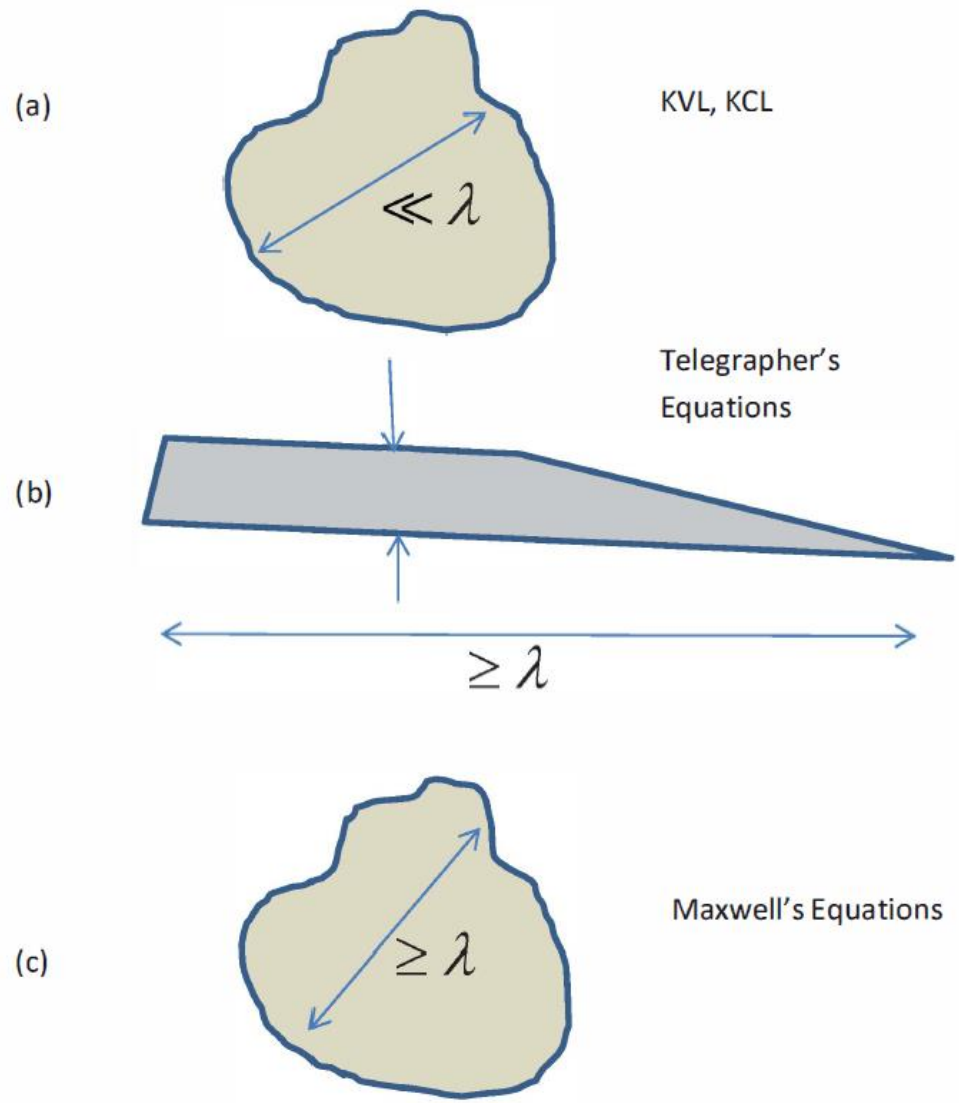
V, I vary
~ useless

- Dependency
  - antenna gain
  - radar cross-section

# Electrical Length

- Behavior (and description) of any circuit depends on his electrical length at the particular frequency of interest
  - E≈0 → Kirchhoff
  - E>0 → wave propagation

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left( \frac{l}{\lambda} \right)$$



# Maxwell's Equations

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J$$

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

- Constitutive equations

$$D = \varepsilon \cdot E$$

$$B = \mu \cdot H$$

$$J = \sigma \cdot E$$

- Vacuum

$$\mu_0 = 4\pi \times 10^{-7}\ H/m$$

$$\varepsilon_0 = 8{,}854 \times 10^{-12}\ F/m$$

$$c_0 = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}} = 2{,}99790 \cdot 10^8\ m/s$$

# Electromagnetic fields with harmonic time dependence

$$X = X_0 e^{j\cdot\omega\cdot t} \qquad \frac{\partial X}{\partial t} = j\cdot\omega\cdot X$$

$$g(\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}dt \qquad f(t) = \int_{-\infty}^{\infty} g(\omega)\cdot e^{j\omega t}d\omega$$

- Maxwell's Equations more simple

$$\nabla^2 E + \omega^2 \varepsilon\mu E = j\omega\mu J + \frac{1}{\varepsilon}\nabla\rho$$

$$\nabla^2 H + \omega^2 \varepsilon\mu H = -\nabla\times J$$

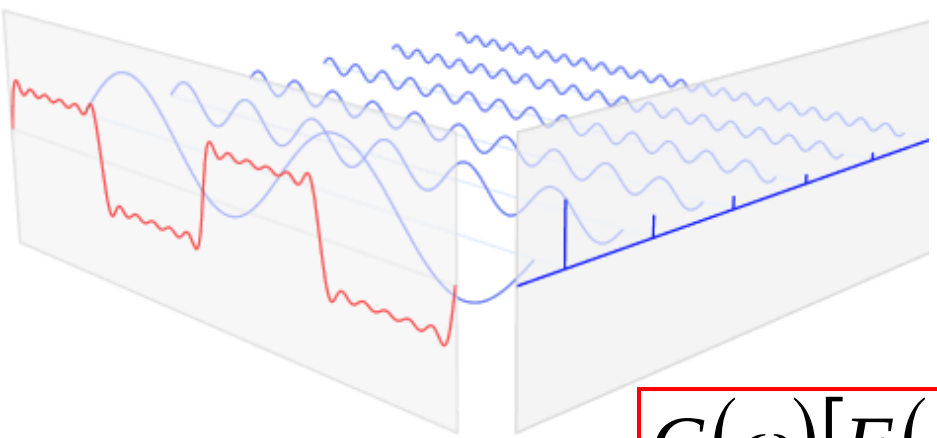
$$\nabla\cdot E = \frac{\rho}{\varepsilon}$$

$$\nabla\cdot H = 0$$

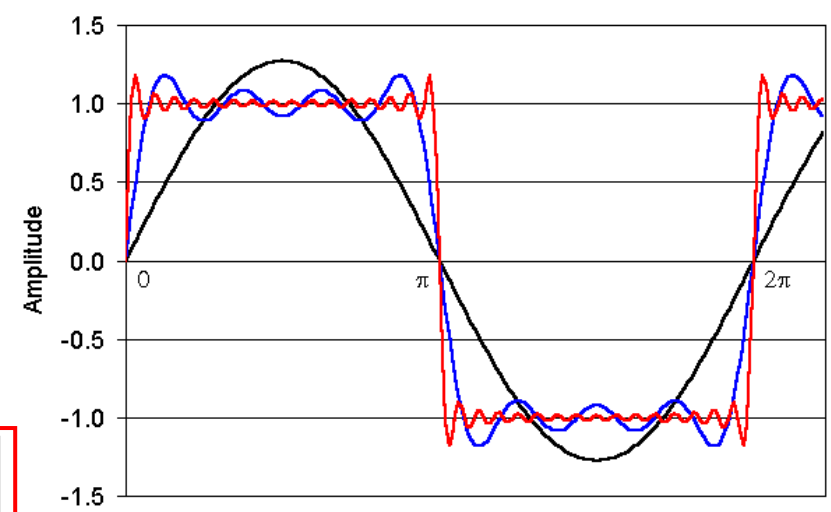
# Mathematical models

- particular cases where analytical solution exists
  - harmonic signals, Fourier Transform, frequency spectrum

$$X = X_0 e^{j\cdot\omega\cdot t} \qquad \frac{\partial X}{\partial t} = j\cdot\omega\cdot X \qquad g(\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}\,dt \qquad f(t) = \int_{-\infty}^{\infty} g(\omega)\cdot e^{j\omega t}\,d\omega$$

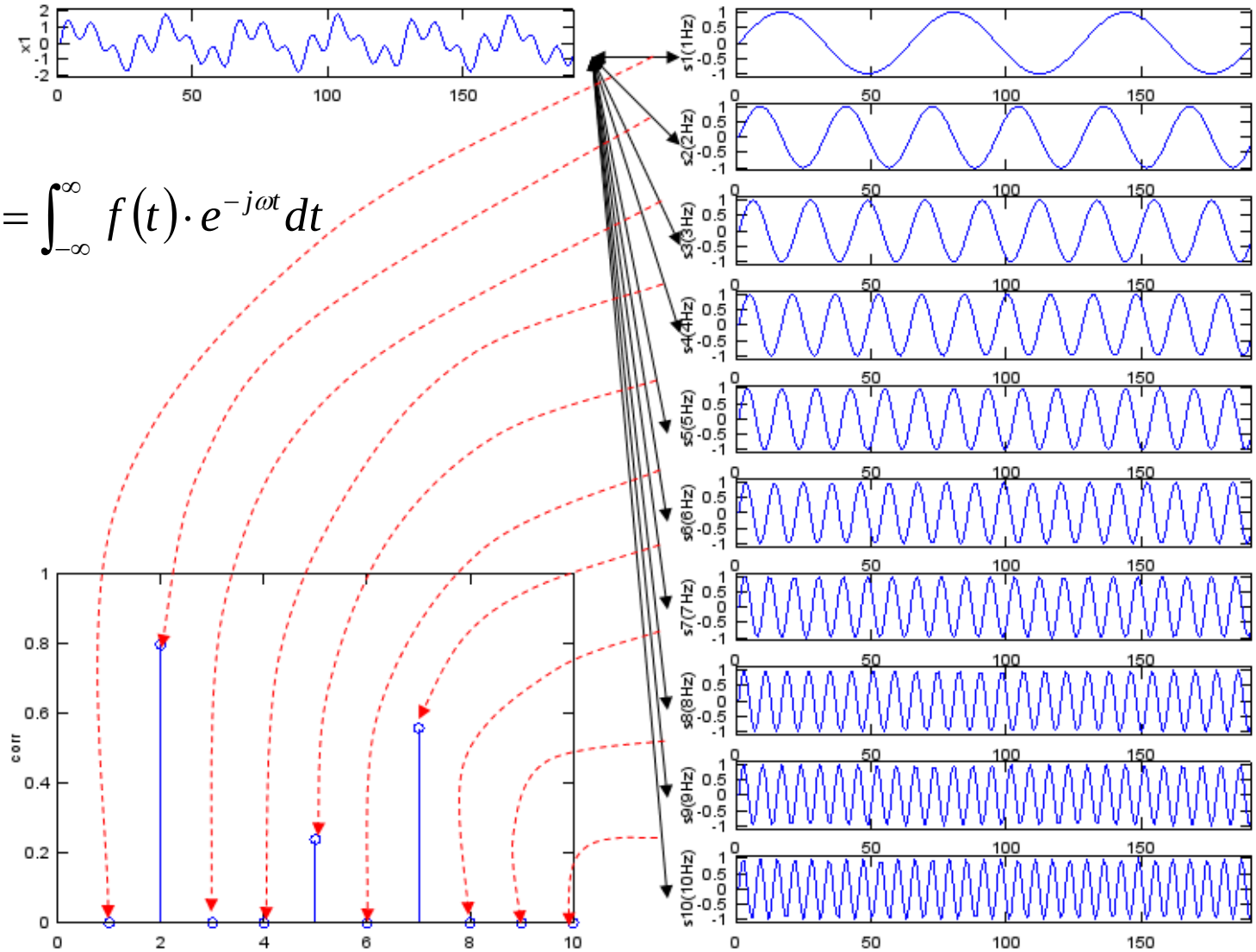


$$\boxed{G(\omega)[F(\omega)]}$$

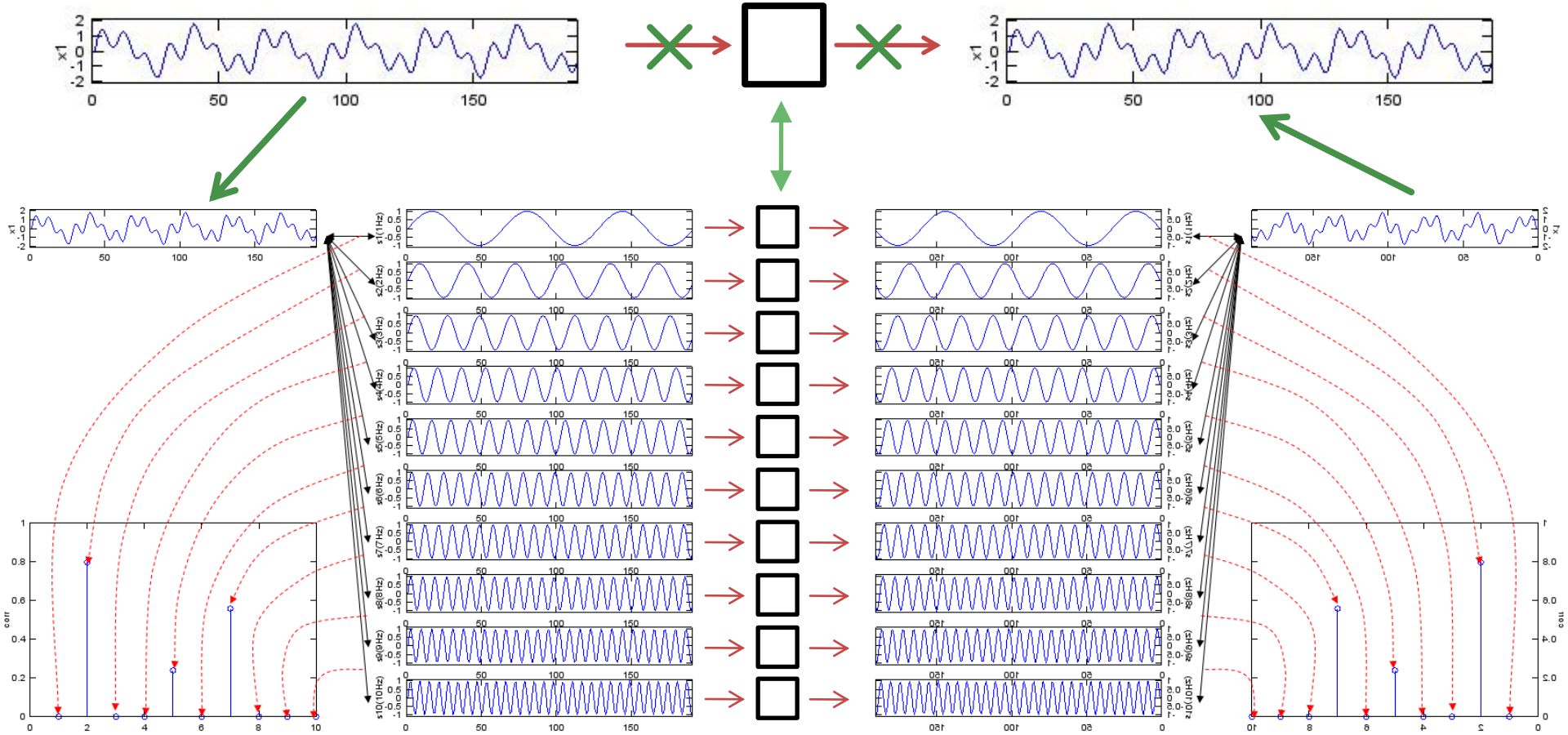


# Mathematical models

$$g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt$$



# Mathematical models




$$F(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}dt$$

$$\boxed{G(\omega)[F(\omega)]}$$

$$g(t)=\int_{-\infty}^{\infty} G(\omega)\cdot e^{j\omega t}d\omega$$

# Wave equations

- Helmoltz equations or Wave equations

Medium void of free electric charges

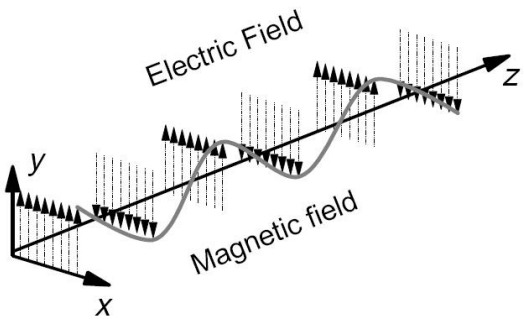
$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

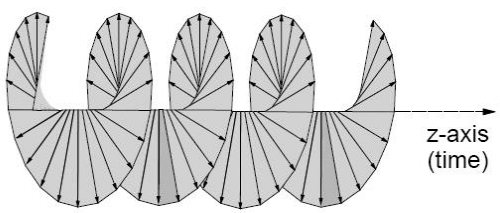
$$\gamma^2 = -\omega^2 \varepsilon \mu + j\omega\mu\sigma$$

$\gamma$ – propagation constant (known also as phase constant or wave number)

# Solutions of the wave equations




Wave Propagation




Circular Polarization

Electric field only in Oy direction, ← through judicious choice
wave traveling after Oz direction ← of the coordinate system

$$E_y = E_+ e^{-\gamma \cdot z} + E_- e^{\gamma \cdot z}$$

$$\gamma = \sqrt{-\omega^2 \varepsilon \mu + j\omega\mu\sigma} = \alpha + j \cdot \beta$$

If we have only the positive direction wave $E_+$=> A

$$E_y = Ae^{-(\alpha + j \cdot \beta) \cdot z}$$

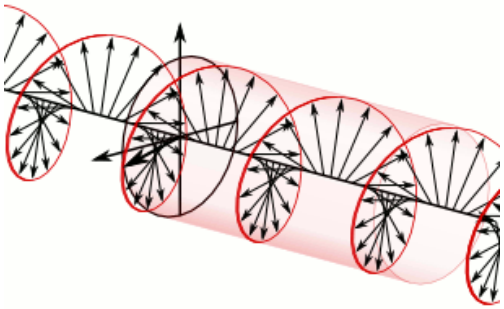
Harmonic Field

$$E_y = A \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$$

Amplitude

Attenuation

Wave Propagation (simultaneous space and time variation)



# Attenuation

$$E_y(z_1) = Ct \cdot e^{-\alpha \cdot z_1} \cdot e^{j(\omega \cdot t - \beta \cdot z_1)} \qquad E_y(z_2) = Ct \cdot e^{-\alpha \cdot z_2} \cdot e^{j(\omega \cdot t - \beta \cdot z_2)}$$

$$W, P \sim \int E^2$$

$$A = \frac{P_2}{P_1} = \frac{Ct^2 \cdot e^{-2\alpha \cdot z_2}}{Ct^2 \cdot e^{-2\alpha \cdot z_1}} = e^{-2\alpha \cdot (z_2 - z_1)}$$

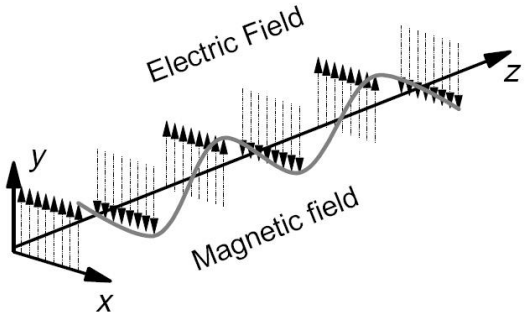
$$A[dB] = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \left[ e^{-2\alpha \cdot (z_2 - z_1)} \right]$$

$$A[dB] = -20 \cdot \alpha \cdot (z_2 - z_1) \log_{10} e = -8.686 \cdot \alpha \cdot (z_2 - z_1)$$

$$A / L[dB / km] = -8.686 \cdot \alpha < 0$$

- Attenuation usually expressed in **dB/km**
  - most of the time a positive value is used
  - "-" sign = **implied** by the word used

# Plane wave parameters




$$\nabla \times E = -j\omega\mu \cdot H$$

$$H_x = \frac{j\gamma \cdot E_y}{\omega\mu}$$

Lossless Medium, σ = 0 $\qquad \gamma = j\omega \cdot \sqrt{\varepsilon\mu}$

$$\eta = \frac{E_y}{H_x} = \sqrt{\frac{\mu}{\varepsilon}}$$ intrinsic impedance of the medium

$$E_y = A \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$$

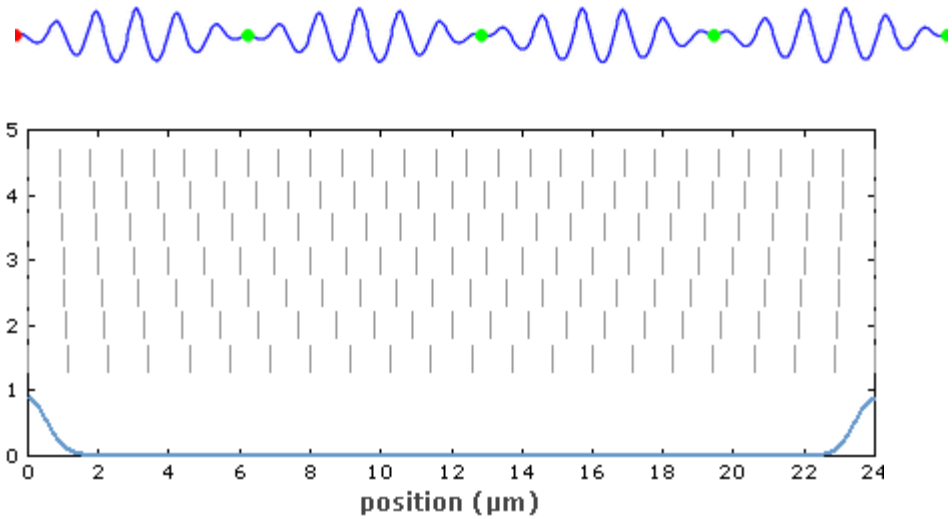
constant phase points: $(\omega \cdot t - \beta \cdot z) = \text{const}$

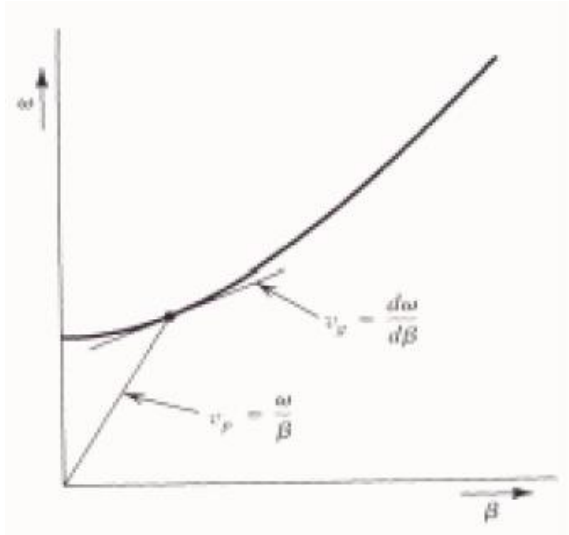
Phase velocity $\qquad v_p = \frac{dz}{dt} = \frac{\omega}{\beta} = \frac{1}{\sqrt{\varepsilon\mu}}$

Group velocity $\qquad v_g = \frac{dz}{dt} = \frac{d\omega}{d\beta}$ $\qquad$ in dispersive media where β = β(ω)

# Group and phase velocities

- Phase velocity – **virtual** speed at which a constant phase point travels (in certain conditions might be greater than the speed of light)
- Group velocity – speed at which the signal (energy, information) propagates (always less or equal to the speed of light in that medium)



# Plane wave parameters

- In vacuum

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega \qquad v = v_g = c_0 \qquad c_0 = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}} = 2{,}99790 \cdot 10^8 \ m/s$$

$$\lambda_0 = \frac{2\pi}{\beta} = \frac{c_0}{f} \qquad T = \frac{2\pi}{\omega} = \frac{1}{f}$$

Space periodicity Time periodicity

- In non-dispersive medium with $\varepsilon_r$

$$c = \frac{1}{\sqrt{\varepsilon \cdot \mu_0}} = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \cdot \mu_0}} = \frac{c_0}{\sqrt{\varepsilon_r}}$$

$n = \sqrt{\varepsilon_r}$ refractive index of a medium $c = \frac{c_0}{n}$

$$T = \frac{2\pi}{\omega} = \frac{1}{f} \qquad \lambda = \frac{2\pi}{\beta} = \frac{c}{f} \qquad \lambda = \frac{c_0}{\sqrt{\varepsilon_r} \cdot f} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

# Solutions of the wave equations

$$E_y = E^+ e^{-\gamma \cdot z} + E^- e^{\gamma \cdot z}$$

Electric field only in Oy direction, ← through judicious choice
wave traveling after Oz direction ← of the coordinate system

$$\gamma = \sqrt{-\omega^2 \varepsilon \mu + j\omega\mu\sigma} = \alpha + j \cdot \beta$$

- wave
  - incident
  - reflected
- wave
  - direct
  - inverse

$$E_y = E^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$$

$$(\omega \cdot t - \beta \cdot z) = \text{const}$$

$$E_y = E^- \cdot e^{\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$(\omega \cdot t + \beta \cdot z) = \text{const}$$

points of constant phase

# Solutions of the wave equations

- wave
  - incident
  - reflected
- wave
  - direct
  - inverse

$$E_y = E^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + E^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$H_z = H^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + H^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$V(z) = V^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + V^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

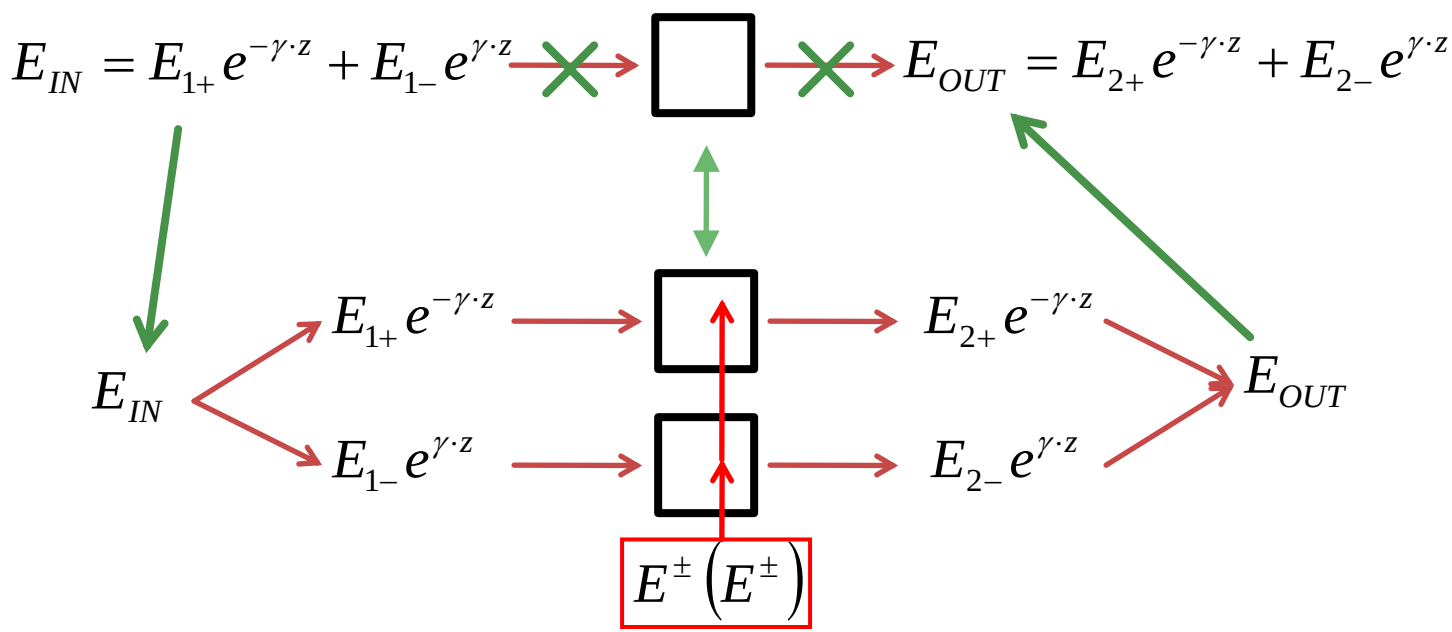
$$I(z) = I^+ \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)} + I^- \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

$$V(z) = V^+ \cdot e^{j(\omega \cdot t - \beta \cdot z)} + V^- \cdot e^{j(\omega \cdot t + \beta \cdot z)}$$

# Mathematical modeling

- particular cases where analytical solution exists
  - wave in a single direction $E^{+}\left(E^{+}\right),\quad E^{-}\left(E^{-}\right)$
  - wave
    - incident
    - reflected
  - wave
    - direct
    - inverse

$$E_y = E^{+}\cdot e^{-\alpha\cdot z}\cdot e^{j(\omega\cdot t-\beta\cdot z)} + E^{-}\cdot e^{-\alpha\cdot z}\cdot e^{j(\omega\cdot t+\beta\cdot z)}$$



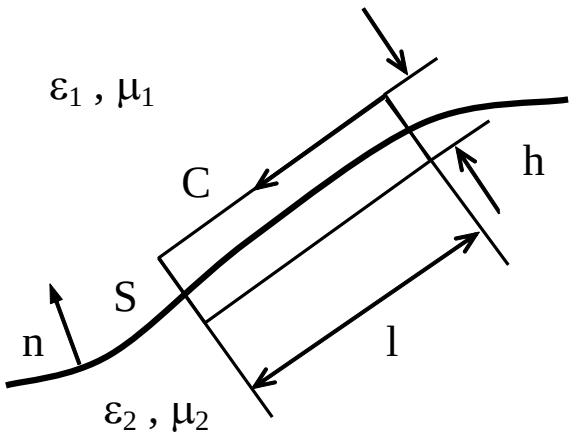
# Modes in delimited media

- Electromagnetic fields with harmonic time dependence
  - Maxwell's Equations simplified

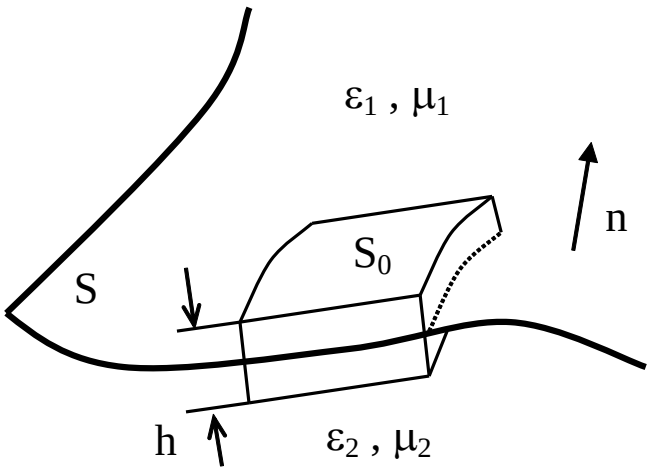
$$X = X_0 e^{j\cdot\omega\cdot t} \qquad \frac{\partial X}{\partial t} = j\cdot\omega\cdot X \qquad g(\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}dt \qquad f(t) = \int_{-\infty}^{\infty} g(\omega)\cdot e^{j\omega t}d\omega$$

- In delimited media the solutions of Maxwell's Equations must also verify boundary conditions
  - solutions must respect some supplemental conditions

# Interface conditions on the interface between two different media




a)




b)

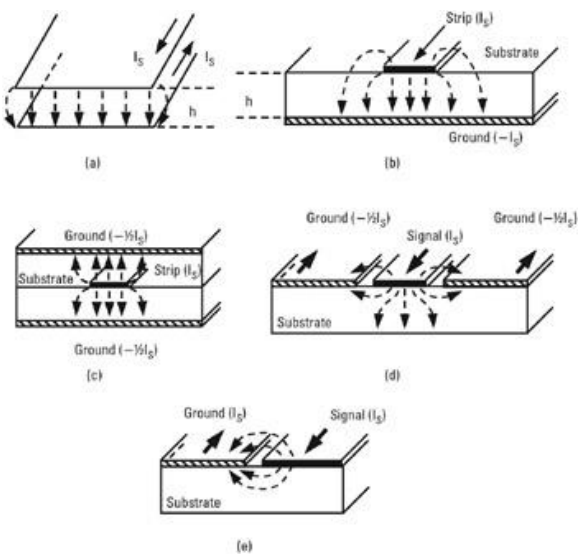
$$n \times (E_1 - E_2) = 0 \qquad n \cdot (D_1 - D_2) = \rho_S$$

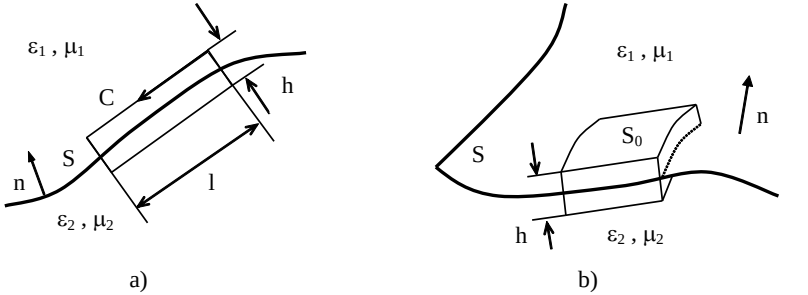
$$n \times (H_1 - H_2) = J_S \qquad n \cdot (B_1 - B_2) = 0$$

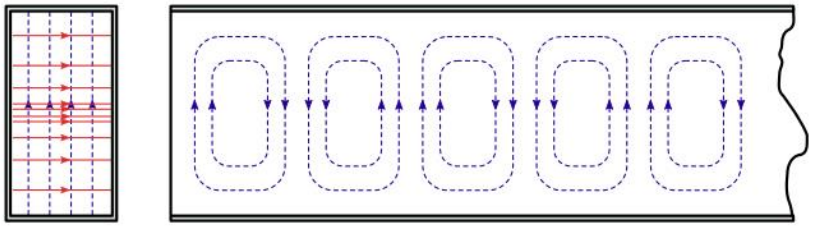
- If one of the media is a perfect conductor (metal) all fields are annulled inside

# Modes in delimited media

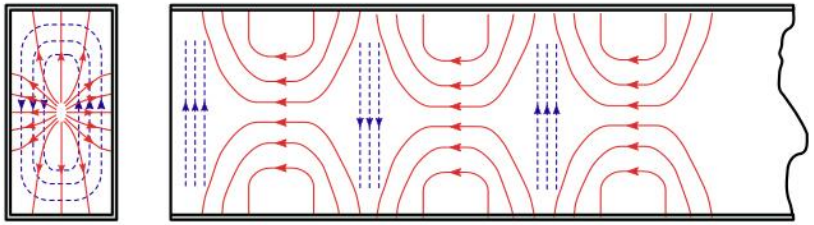
- Electric field must always be normal on an electric wall or annulled
- Magnetic field must always be tangent to an electric wall or annulled



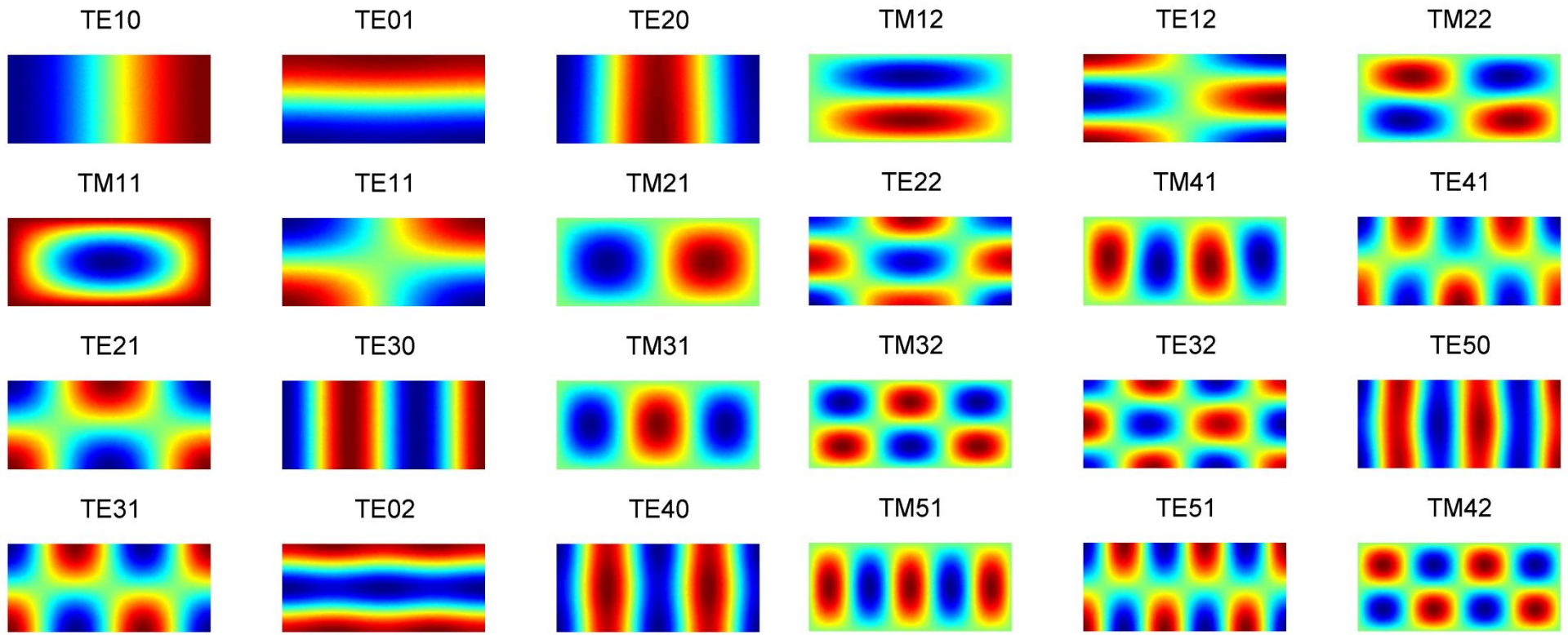



$TE_{10}$



$TM_{11}$

# Moduri in medii delimitate



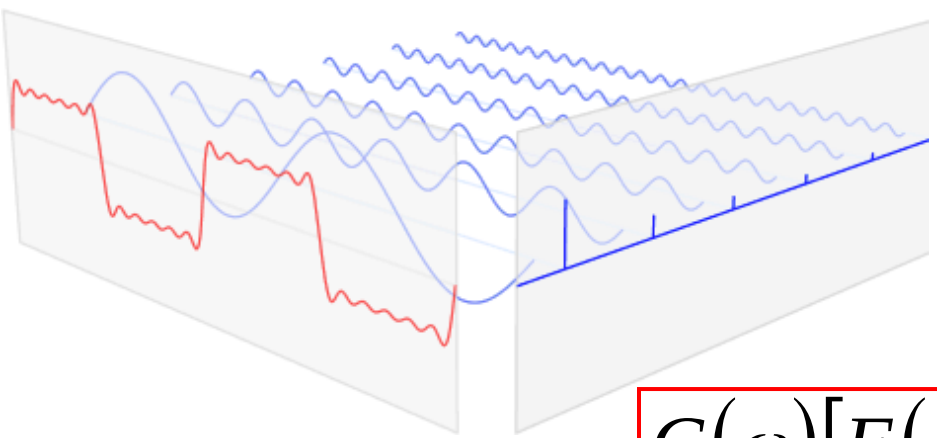

- Similar with Fourier Transform $g(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}dt \qquad f(t)=\int_{-\infty}^{\infty} g(\omega)\cdot e^{j\omega t}d\omega$

$$E^{+},E^{-}=\sum_{1}^{\infty} A_i\cdot Mod_i \qquad A_i=\langle E, Mod_i\rangle$$

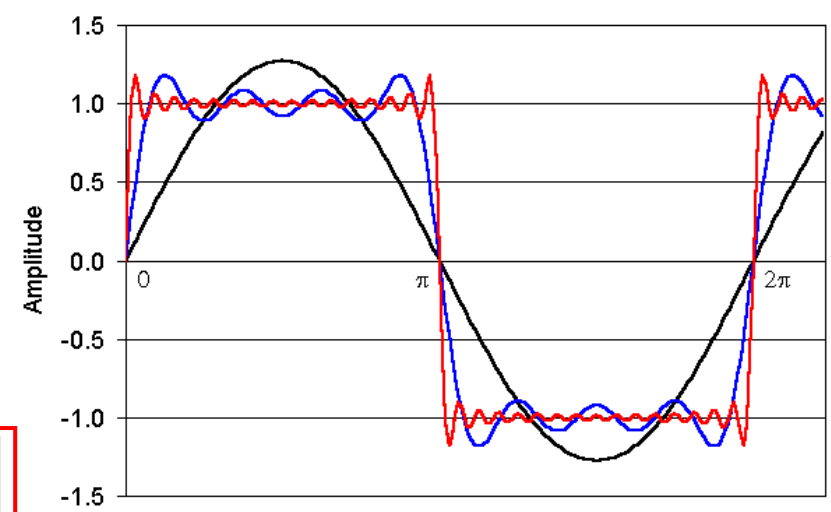
# Mathematical modeling

- particular cases where analytical solution exists
  - harmonic signals, Fourier Transform, frequency spectrum

$$X = X_0 e^{j\cdot\omega\cdot t} \qquad \frac{\partial X}{\partial t} = j\cdot\omega\cdot X \qquad g(\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}\,dt \qquad f(t) = \int_{-\infty}^{\infty} g(\omega)\cdot e^{j\omega t}\,d\omega$$

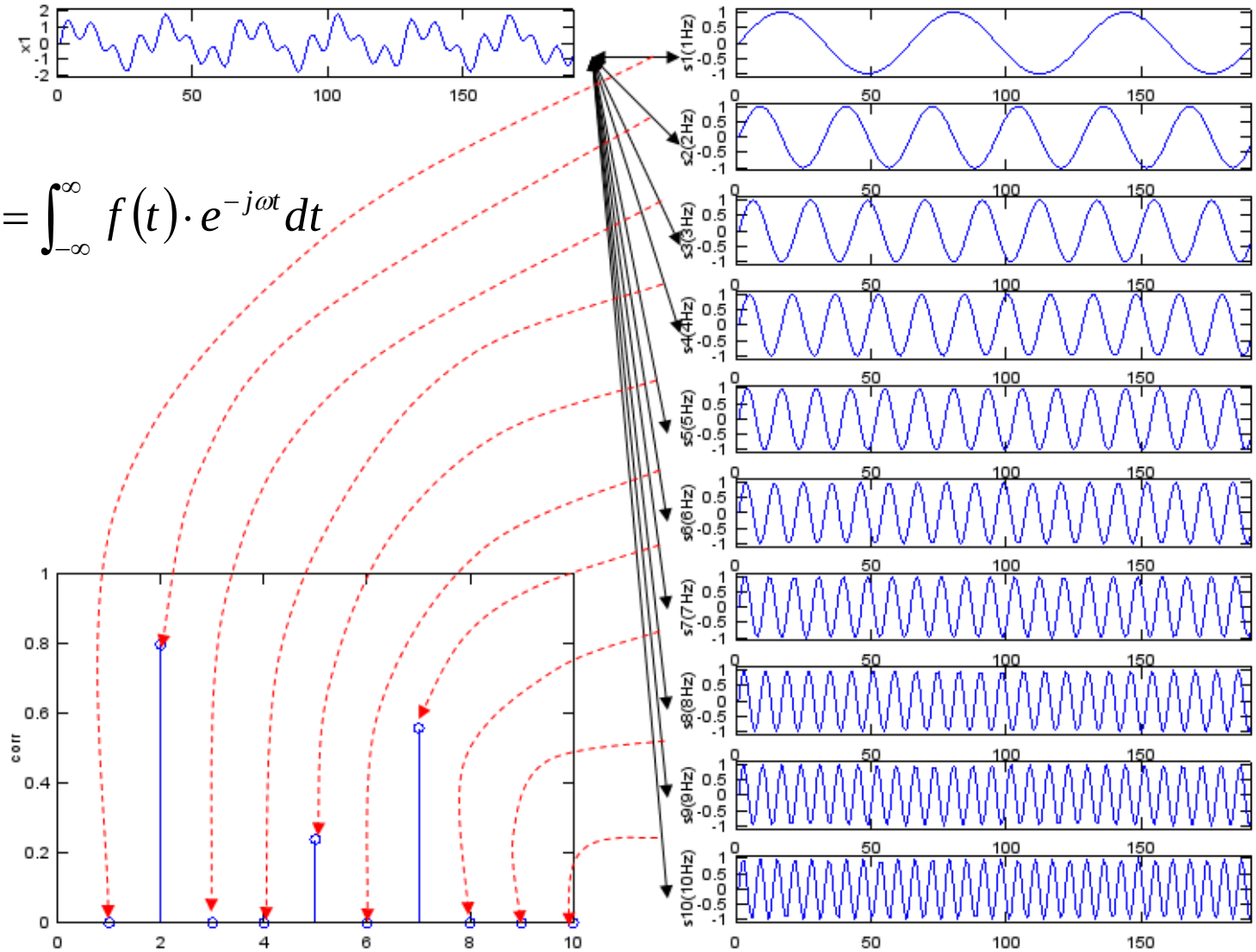


$$\boxed{G(\omega)[F(\omega)]}$$

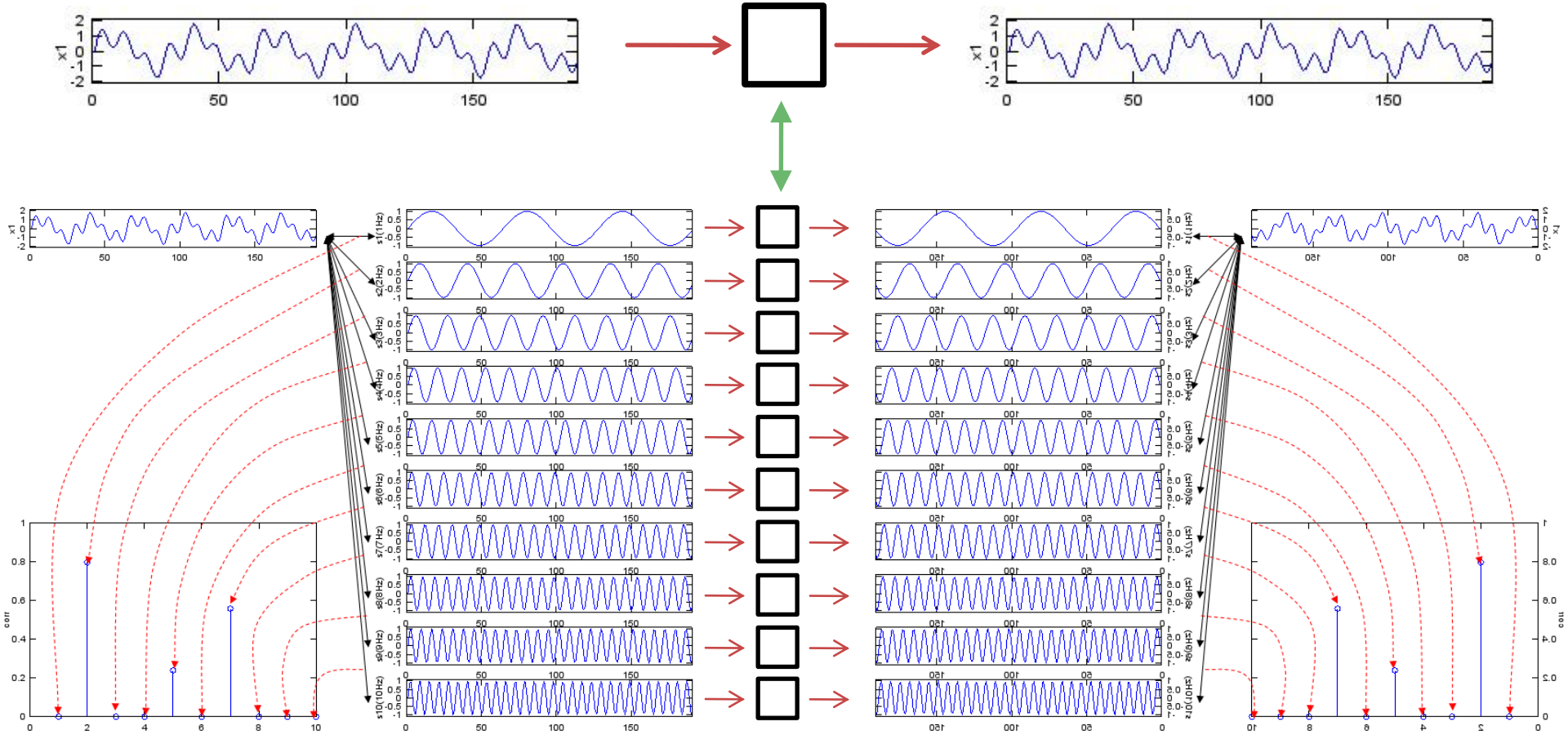


# Mathematical modeling

$$g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt$$



# Mathematical modeling




$$F(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}dt$$

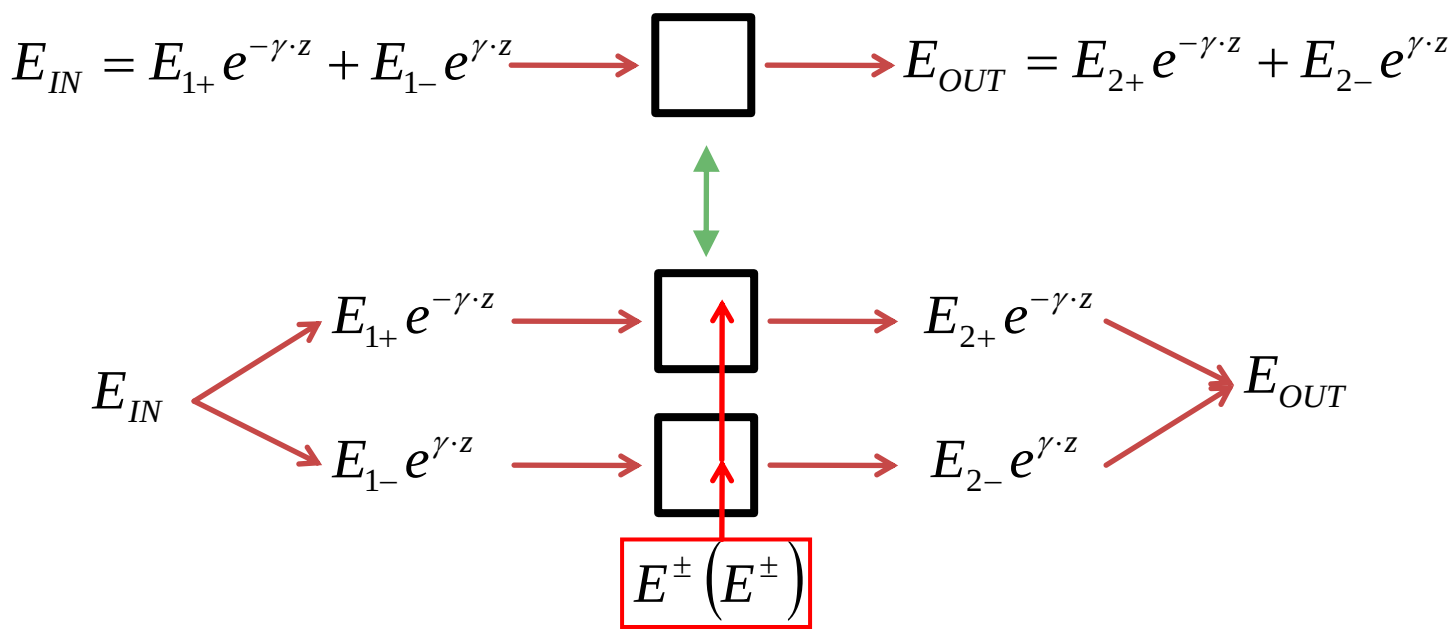
$$\boxed{G(\omega)[F(\omega)]}$$

$$g(t)=\int_{-\infty}^{\infty} G(\omega)\cdot e^{j\omega t}d\omega$$

# Mathematical modeling

- particular cases where analytical solution exists
  - wave in a single direction $E^{+}\left(E^{+}\right),\quad E^{-}\left(E^{-}\right)$
  - wave
    - incident
    - reflected

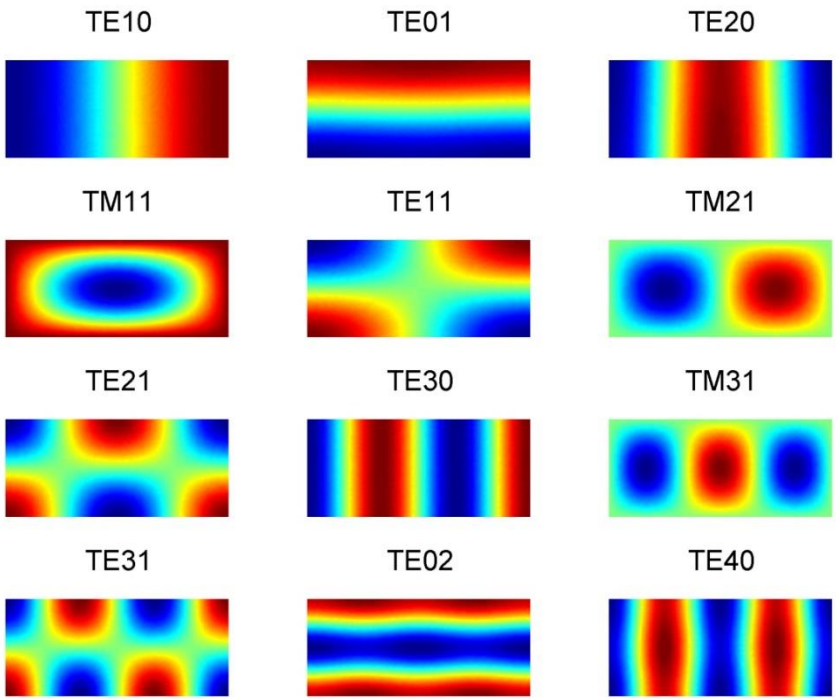
$$E_y = E^{+}\cdot e^{-\alpha\cdot z}\cdot e^{j(\omega\cdot t-\beta\cdot z)} + E^{-}\cdot e^{-\alpha\cdot z}\cdot e^{j(\omega\cdot t+\beta\cdot z)}$$



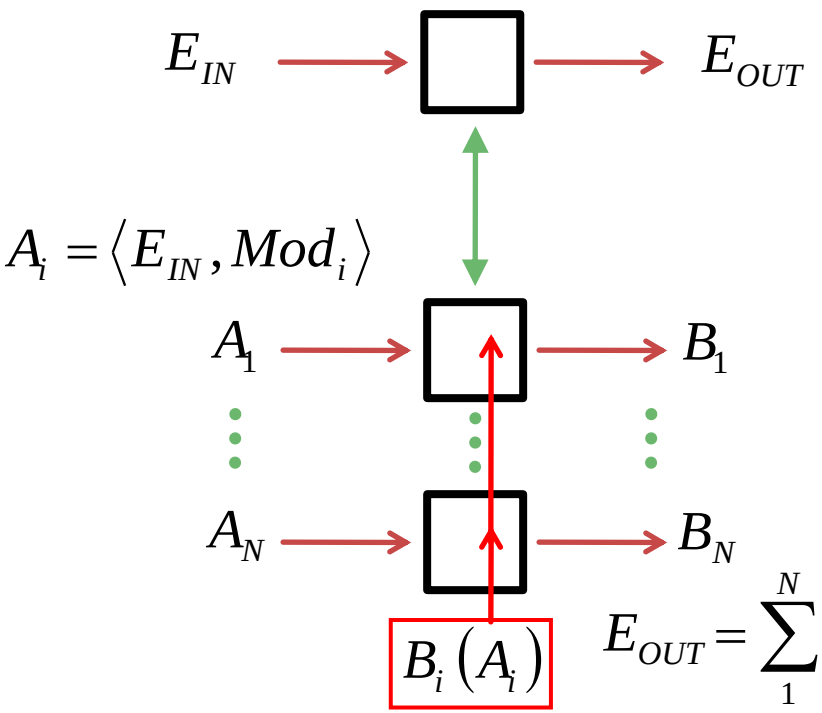

  - wave
    - direct
    - inverse

# Mathematical modeling

- particular cases where analytical solution exists
  - modes in delimited media $B_i(A_i)$




$$E = \sum_{1}^{\infty} A_i \cdot Mod_i \qquad A_i = \langle E, Mod_i \rangle$$




$$E_{OUT} = \sum_{1}^{N} B_i \cdot Mod_i$$

# Contact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